Contents

Overview document for: A weight function theory of basis function interpolants and smoothers.

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ABSTRACT. This document is a brief overview of two documents which continue to develop the weight function theory of basis function smoothers and interpolants. One document considers the zero order theory and one considers the positive order theory.

0.1. Change register.

05/Jul/2007 Created this document using the abstracts from the version 1 documents.

22/Oct/2007 Altered this document using the abstracts from the consolidated version 2 documents i.e. version 2 of arXiv:0708.0780 and version 2 of arXiv:0708.0795.

1. Overview

This work currently consists of **two documents** which continue to develop the **Light weight function theory of basis function smoothers and interpolants**. One document considers the **zero order theory** and one considers the **positive order theory**.

In brief, some important general features:

(1) Extends the positive order work of Light and Wayne [**2**], [**3**] and [**1**] to the zero order case and extends the positive order case to tensor product weight functions.

For both the positive and zero order cases:

(2) A weight function is first defined and then used to define a continuous basis function and a data function Hilbert space are defined using the Fourier transform. This technique is illustrated by several examples.

(3) The standard minimal norm and seminorm interpolants are defined and pointwise orders of convergence are derived on a bounded set.

(4) We define the well known variational non-parametric smoother which stabilizes the interpolant using a smoothing parameter - I call this the Exact smoother. Orders of uniform pointwise convergence are derived on a bounded open set.

(5) A scalable smoother is derived which I call the Approximate smoother. Orders of uniform pointwise convergence are derived on a bounded open set.

(6) For the **zero order** case numeric examples are given which compare the theoretical and actual errors w.r.t. the data function.

2. Zero order document (arXiv:0708.0780)

Here is a short description of the document (the abstract).

In this document I develop a weight function theory of zero order basis function interpolants and smoothers.

In **Chapter 1** the basis functions and data spaces are defined directly using weight functions. The data spaces are used to formulate the variational problems which define the interpolants and smoothers discussed in later chapters. The theory is illustrated using some standard examples of radial basis functions and a class of weight functions I will call the tensor product extended B-splines.

In **Chapter 2** the theory of Chapter 1 is used to prove the pointwise convergence of the minimal norm basis function interpolant to its data function and to obtain orders of convergence. The data functions are characterized locally as Sobolev-like spaces and the results of several numerical experiments using the extended B-splines are presented.

In **Chapter 3** a large class of tensor product weight functions will be introduced which I call the central difference weight functions. These weight functions are closely related to the extended B-splines and have similar properties. The theory of this document is then applied to these weight functions to obtain interpolation convergence results. To understand the theory of interpolation and smoothing it is not necessary to read this chapter.

In **Chapter 4** a non-parametric variational smoothing problem will be studied using the theory of this document with special interest in its order of pointwise convergence of the smoother to its data function. This smoothing problem is the minimal norm interpolation problem stabilized by a smoothing coefficient.

In **Chapter 5** a non-parametric, scalable, variational smoothing problem will be studied, again with special interest in its order of pointwise convergence to its data function. We discuss the *SmoothOperator* software (freeware) package which implements the Approximate smoother algorithm. It has a full user manual which describe several tutorials and data experiments.

3. Positive order document (arXiv:0708.0795)

Here is a short description of the document (the abstract).

In this document I develop a weight function theory of positive order basis function interpolants and smoothers.

In **Chapter 1** the basis functions and data spaces are defined directly using weight functions. The data spaces are used to formulate the variational problems which define the interpolants and smoothers

discussed in later chapters. The theory is illustrated using some standard examples of radial basis functions and a class of weight functions I will call the tensor product extended B-splines.

Chapter 2 shows how to prove functions are basis functions without using the awkward space of test functions So,n which are infinitely smooth functions of rapid decrease with several zero-valued derivatives at the origin. Worked examples include several classes of well-known radial basis functions.

The goal of **Chapter 3** is to derive 'modified' inverse-Fourier transform formulas for the basis functions and the data functions and to use these formulas to obtain bounds for the rates of increase of these functions and their derivatives near infinity.

In **Chapter 4** we prove the existence and uniqueness of a solution to the minimal seminorm interpolation problem. We then derive orders for the pointwise convergence of the interpolant to its data function as the density of the data increases.

In **Chapter 5** a well-known non-parametric variational smoothing problem will be studied with special interest in the order of pointwise convergence of the smoother to its data function. This smoothing problem is the minimal norm interpolation problem stabilized by a smoothing coefficient.

In **Chapter 6** a non-parametric, scalable, variational smoothing problem will be studied, again with special interest in its order of pointwise convergence to its data function.

Bibliography

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